**Homework 04: Fundamental Concepts IV**

**PHYS550 – Quantum Mechanics I**

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**September 22-, 2021**

***Additional Texts Referenced: Introduction to Quantum Mechanics, Griffiths and Schroeter***

**Problem 1.28**

*Construct the transformation matrix that connects the Sz diagonal basis to the Sx diagonal basis. Show that your result is consistent with the general relation*



Since we are told to construct the matrix without *using* the general relation, we turn to the matrix definition:



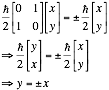
We know it’s 2x2 since our operators are for spin, which are 2x2 on their own. By the matrix definitions, uij is *.* Let Sz be the “a” operator and Sx be the “b” operator. Each will have two “vectors” in them corresponding to spin up and spin down.



*,*



ADDENDUM: this problem was done before class on Friday, wherein it was revealed that the eigenvectors for Sx needed to be derived. Well, we already know the eigenvalues need to be ±ћ/2. Using the known Sx matrix we get the following relation:



By normalization, we know this means y and x have to have magnitude 1/√2. Setting x to 1/√2 provides the two desired eigenvectors:



And so from the definition for U we obtain:

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Which is, in fact, unitary. (And it contains no imaginary numbers and thus is its own hermitian conjugate.) A simple way to test if this works is to feed a test vector in, say [0, 1], and see if it converts.

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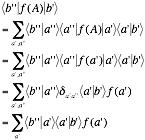
Which satisfies theorem 3 (1.157) completely. Now we use the outer product definition to confirm.



Which is exactly the same, and thus our work is done.

**Problem 1.29**

*a) Suppose that f(A) is a function of a Hermitian operator A with the property . Evaluate when the transformation matrix from the a’ basis and the b’ basis is known.*



Using the complicated one and the property of the dirac delta to change all a’’ to a’. (since it’s zero whenever it isn’t.) However, we cannot reduce the 1 at the end since f(a’) depends on a’ and could be anything due to the function.

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*b) Using the continuum analogue of the result obtained in a), evaluate*



*Simplify your expression as far as you can. Note that r is √(x2+y2+z2), where x, y, and z are* ***operators****.*

First of all, the continuum result can be written as:



The question becomes what exactly a’ is in this case? b translates directly to momentum. a’ has to be an eigenvalue (or eigenfunction) such that *.* This would only work if the kets were in the r basis. So we can rewrite as:



Note that we are in d3r’ since the bolded momentum values imply this is a three-dimensional situation. Luckily the function itself is just a function of r alone. We note that the r’ eigenvalues are just the observed values of r, so we’re integrating over the variable r. As suggested, we convert to xyz.



NOTE: I do not think this is correct. I can’t tell you *why* it’s incorrect since we’re integrating over three things that are effectively numbers so integration by parts should work just fine. I do know that the other students and I have been trying to work out what to do for quite some time and nobody’s gotten anything that makes sense. (Trying to replace r with the definition of r didn’t go anywhere either.) So I’m submitting this and waiting for the homework explanations to be published.

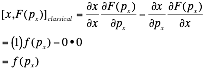
**Problem 1.30**

*a) Let x and px be the coordinate and linear momentum in one dimension. Evaluate the classical Poisson bracket [x,F(px)]classical.*

The classical poisson bracket is defined by 1.230:



Since our given equation is only concerned with x, the sum goes away when we perform the substitution:



Where f(px) is the derivative of F(px) with respect to px.

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| f(px) |

*b) Let x and px be the corresponding quantum mechanical operators this time. Evaluate the commutator [x,exp(ipxa/ћ)].*

Using part a) above we note that the Poisson Bracket is f(px) or ia/ћ exp(ipxa/ћ). Now we use equation 1.229 which correlates the Poisson Bracket to the commutator with a factor of iћ. This makes the commutator:

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| -a exp(ipxa/ћ). |

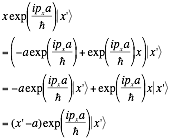
The quickness of this solution is suspicious, but attempts to evaluate the commutator directly via a test function proved fruitless.

*c) Using the result obtained in b) prove that the following is an eigenstate of the coordinate operator x.*



*What is the corresponding eigenvalue?*

Well let’s just see what happens when we tack x onto the end of the expression.



This proves the state is an eigenstate because we ended with the original eigenstate times a constant—(x’-a).

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| Eigenvalue: x’-a |

(The step that used part b) made use of the fact [A,B] = AB-BA => AB=[A,B]+BA)